Enrollment No:-__________ Exam Seat No:-_______

C.U.SHAH UNIVERSITY

Summer-2015

Subject Code: 4TE04DSM1 **Subject Name: Discrete Mathematics**

Course Name: B.Tech(CE,IT) Date: 19/5/2015

Semester: IV Marks: 70

Time: 02:30 TO 05:30

Instructions:

1) Attempt all Questions in same answer book/Supplementary.
2) Use of Programmable calculator & any other electronic instrument prohibited.
3) Instructions written on main answer book are strictly to be obeyed.
4) Draw neat diagrams & figures (if necessary) at right places.
5) Assume suitable & perfect data if needed.

SECTION – I

Q-1 (a) Find the least and greatest element in the POSET $\langle Z^{+}, D\rangle$, if they exist. [01]

(b) Define: Atom and find all atoms of Boolean algebra $\langle S_{30}, D\rangle$. [02]

(c) Symbolize the expression: [02]

i) "If x is odd and x is perfect square then x is divisible by 3."

ii) "All birds can fly."

(d) Over the universe of animals: [02]

$A(x)$: x is a whale, $B(x)$: x is a fish, $C(x)$: x lives in water

Translate the following in your own words.

i) $(\exists x)\big(B(x)\wedge \sim A(x)\big)$ ii) $(\forall x)\big(A(x)\vee C(x)\big)\rightarrow B(x)$

Q-2 (a) Let $A=\{1,2,3,4,5,6\}$ along with partial ordered relation D such that aDb means "a divides b" then [05]

i) Find cover of each element and draw the Hasse diagram.
ii) Find greatest element, least element, minimal element and maximal element, if they exist.
iii) Find the greatest lower bound and least upper bound of {2}, if they exist.

(b) Let $\langle L,\leq\rangle$ be a lattice $a,b\in L$,prove that $a\leq b\Leftrightarrow a*b=a\Leftrightarrow a\oplus b=b$. [05]

(c) If $\langle L, *, \oplus, ', 0, 1\rangle$ is a complemented and distributive lattice , prove that $(a*b)' = a' \oplus b'$. [04]

OR

Q-2 (a) Let $A = \{2, 3, 4, 9, 12, 18\}$ along with partial ordered relation D such that aDb means "*a* divides *b*" then [05]

i) Find cover of each element and draw the Hasse diagram.
ii) Find greatest element, least element, minimal element and maximal element, if they exist.
iii) Find the greatest lower bound and least upper bound of {4,12}, if they exist.

(b) Prove that $\langle P(\{a, b, c\}), \subseteq\rangle$ is a lattice and hence find the complement of each element, if exists. [05]

(c) Prove that $\langle P(X), \subseteq\rangle$ is isomorphic to $\langle L^2, \leq\rangle$. Where $X = \{a, b\}$, $L = \{0, 1\}$ and "$\leq$"- is usual partial order relation. [04]

Q-3 (a) Answer the following: [10]

i) Let $\langle B, *, \oplus, ', 0, 1\rangle$ be a Boolean algebra and $a, b, c \in B$, show that $a*(a' \oplus b) = (a*b) \oplus (c*b*a) = a*b$.

ii) Obtain the SOP canonical form of the Boolean expression in three variables $(x_1 \oplus x_2)' \oplus (x_1' * x_3)$.

iii) Obtain the POS canonical form of the Boolean expression in three variables $(x_1 * x_3) \oplus (x_1' * x_2) \oplus (x_2 * x_3)$.

(b) Answer the following: [04]

i) Examine the validity of the $R \vee S$ follows logically from the premises $C \vee D, (C \vee D) \rightarrow \square H, \square H \rightarrow (A \wedge \square B)$ and $(A \wedge \square B) \rightarrow (R \vee S)$.

ii) Show that $\exists y \forall x P(x, y) \Rightarrow \forall x \exists y P(x, y)$.

OR

Q-3 (a) State and prove Stone's representation theorem. [10]

(b) Show that $\square r$ is a valid conclusion from the premises $p \rightarrow \square q, r \rightarrow p$ and q [04]

i) with truth table ii) without truth table

SECTION-II

Q-4 (a) State Pigeonhole principle. [01]

(b) How many edges are there in undirected graph with 6 vertices each of degree 5? [02]

(c) Give an example of a monoid which is not a group. Justify your answer. [02]

(d) How many 5-digit even numbers can be formed using the digits 1, 2, 3, 4, 5 once? [02]

Q-5 (a) State and prove Cayley's theorem on group. [07]

(b) Show that $\langle Z_6, +_6 \rangle$ is isomorphic to $\langle Z_7^*, \times_7 \rangle$. [07]

OR

Q-5 (a) Answer the following: [07]

i) State and prove Lagrange's theorem on group.

ii) Show that the kernel of homomorphism $g : \langle G, * \rangle \to \langle H, \Delta \rangle$ is a subgroup of a group $\langle G, * \rangle$.

(b) Let $S = \{1, 2, 3\}$ and S_3 be the set of permutation on S. Find all proper subgroups of a group $\langle S_3, \Diamond \rangle$ and identify that which subgroup is normal? Where $\Diamond$ represents composition of two permutations. [07]

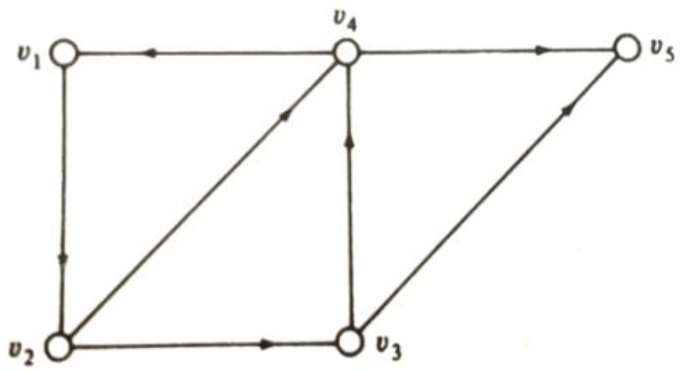
Q-6 (a) Give three different representations of a tree from [07]

$$\left(v_0\left(v_1\left(v_2\right)\left(v_3\left(v_4\right)\left(v_5\right)\right)\right)\left(v_6\left(v_7\left(v_8\right)\right)\left(v_9\right)\left(v_{10}\right)\right)\right)$$

Also identify root, branch nodes and leaf nodes from the tree.

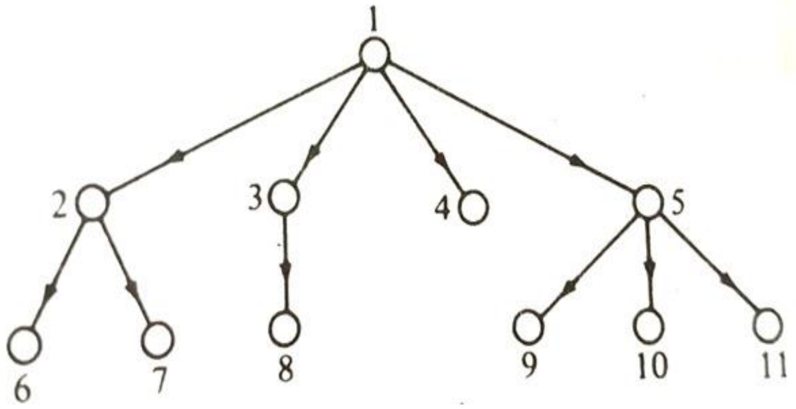
(b) From the graph given below, answer the following: [07]

1. Find in degree, out degree and total degree of each vertex.
2. Find reachable set of each vertex.
3. Find all node bases.
4. Find all strong components.
5. Write the adjacency matrix from the given digraph.



OR

Q-6 (a) Obtain binary tree equivalent to the tree given below: [07]



(b) Answer the following: [07]

i) Let $E=\{a,b,c,d,e\}$,

$$\underset{\sim}{A}=\{(a,0.3),(b,0.8),(c,0.5),(d,0.1),(e,0.9)\},$$

$$\underset{\sim}{B}=\{(a,0.7),(b,0.6),(c,0.4),(d,0.2),(e,0.1)\}$$

Find the following:

1) $\underset{\sim}{A}\cup\underset{\sim}{B}$ 2) $\underset{\sim}{A}\cdot\underset{\sim}{B}$ 3) $\underset{\sim}{A}\,\hat{+}\,\underset{\sim}{B}$ 4) $\underset{\sim}{A}-\underset{\sim}{B}$

ii) State De Morgan's Laws for fuzzy subsets and prove any one.